Proceedings of the 2017 Winter Simulation Conference
W. K. V. Chan, A. D'Ambrogio, G. Zacharewicz, N. Mustafee, G. Wainer, and E. Page, eds.

CALIBRATION OF A STOCHASTIC AGENT-BASED MODEL FOR RE-HOSPITALIZATION NUMBERS OF PSYCHIATRIC PATIENTS

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ABSTRACT

Calibration is a vital part of the modeling and simulation process and denotes the determination of parameter values by estimating them from comparison between simulation results with reference data. During the last decades a lot of algorithms have been developed for this purpose which are able to fit mentioned parameter values generically without information about the modeled system or the simulation. Especially for stochastic simulation models these routines very often require thousands of iterative simulation executions which, in case of large agent-based models, might be too time intensive.

In this paper we illustrate a real world example for such a problem and present a solution for it based on probability theory. Hereby we not only calibrate the desired parameters, but also find a measure for the quality of the fit as well. By presenting this example we want to motivate modelers to analyze agent-based models to save costly calibration time.

1 INTRODUCTION

Identification of parameter values or, probably more often used, calibration is a vital part of the modeling and simulation process. Hereby we denote determination of parameter values without measuring or calculating them directly from the real system, but estimating them by comparing simulation results with reference data. As one cannot simply "invert" a simulation model to map output onto parameters, a highly complex optimization problem results: Find a set of parameter values so that the difference between simulation output and reference data is as small as possible. In order to solve this problem so called calibration algorithms, usually iterative meta-heuristics, have to be used. This process is sketched in Figure 1.

In the present work we want to describe a calibration process which was necessary to fit specific parameters in a simulation model for re-hospitalizations of psychiatric patients in Austria, Italy (the region of Veneto, to be specific) and Slovenia. Hereby re-hospitalization addresses the voluntary or involuntary readmission

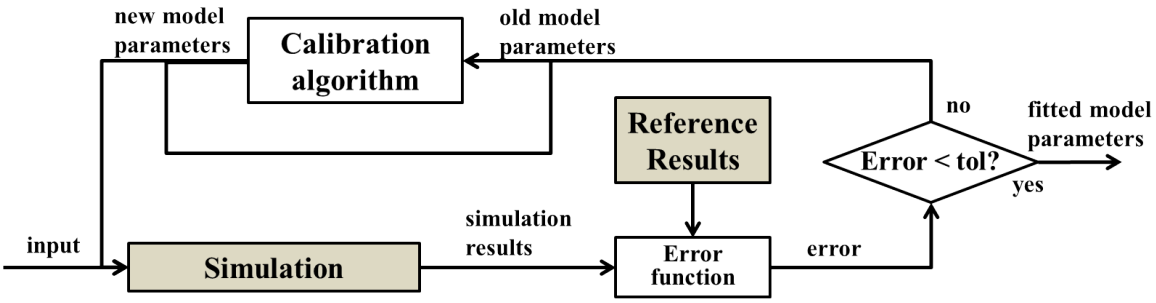

Figure 1: Sketch of the use of a calibration algorithm in the loop. Starting from an initially guessed parameter value the simulation is executed in the loop. After comparison with reference results, usually some validation data, an error between simulation results and reference is calculated. This error decides about whether it is necessary to readjust the parameter values using a calibration routine or they are sufficiently fitted.

of a psychiatric patient to a hospital after a certain period of time. The prediction of the total number of re-hospitalizations per country until 2040 was the main objective of the model. Three different synthesing modeling tasks with several sub-scenarios had to be investigated which are in detail explained in section 2.

For its flexibility an agent-based approach was chosen to tackle the three modeling tasks. A detailed definition of the agent-based model (ABM) is given in section 3. While the first scenario, a base-scenario, could be modeled and parametrized directly, the other two scenarios required calibration of specific parameter values. Hereby we especially faced two difficulties which distinguished this calibration task from most others:

- **Stochasticity.** The ABM is stochastic. Hence there is no unique mapping from parameters to output of the simulation.
- **Simulation Time.** The ABM is huge (up to 9 Million agents) and, though parallelized, requires several minutes to be finished.

Although there are algorithms specifically developed/adapted for stochastic optimization problems (see section 4) none of these could be applied to our case as they require thousands of simulation executions until the parameters are sufficiently fitted. As a consequence we analyzed the stochastic basics of the model and found a much more elegant way to fit the parameters. This strategy is focus of the present work and is presented in Section 5.

2 USE CASE AND SCENARIOS

2.1 Cephos-Link

Re-hospitalization rates of psychiatric patients are considered as a metric of quality of care. In the EU - FP7 project Cephos-Link (**C**omparative **E**ffectiveness research on **P**sychiatric **HOS**pitalisation by record **LINK**age of large administrative data sets) the focus lay on collecting and analyzing re-hospitalization patterns of psychiatric patients which are at least 18 years old with an ICD10 diagnosis F2-F6 (mainly schizophrenia and bipolar disorders), divided into the group of psychotic and non-psychotic diseases, of the six participating European countries (Austria, Finland, Italy, Norway, Romania, Slovenia).
One of the main objectives was to compare different types of health service interventions in terms of differences in re-hospitalization outcomes (Kalseth, Lassemo, Wahlbeck, Haaramo, and Magnussen 2016). Therefore in the first phase the data of the six countries was pooled and analyzed using statistical methods like logistic and Cox regression models (Urach, Zauner, Wahlbeck, Haaramo, and Popper 2016). Afterwards we used the results together with expert knowledge to define questions about the impact of identified risk

factors which enforce re-hospitalizations. Three of these questions were formulated in standardized HTA format to be answered by a dynamic simulation model and resulted in three modeling tasks (see below). Overall, the results of the project should help decision makers to plan and optimize interventions for improving the treatment for psychiatric patients. One key aspect is the preplanning of ressources for the next decades.

2.2 Task A: "Base-Case-Scenario"

One of the main findings of the pooled data was the influence of age and sex on both index hospitalizations as well as re-hospitalizations. Additionally there is an age- and gender- adjusted effect of the diagnosis group. For planning of psychiatric care it is essential to have an estimate of the burden of the disease in the near future.

As a result the first defined task considered development of a simulation model that is, first, able to depict the present situation in Austria, Slovenia and the region of Veneto in Italy and, second, make prognosis until 2040 under assumption that the present hospitalization trends remain valid for changing demographics. Main objective of this simulation task was the correct simulation of aggregated index-hospitalization and re-hospitalization numbers. Present trends were evaluated investigating collected data for 2006 (Austria) and 2013 (Slovenia and Veneto). We will henceforth refer to these dates as "index-years" as they pose for the start-year in each of the simulations. No further sub-scenarios were defined.

2.3 Task B: "Driving Distance Influence"

Moreover re-hospitalization rates in the different NUTS3-regions (**N**omenclature des **U**nits **T**erritoriales **S**tatistiques level 3, small regions) could be observed to vary heavily and studies (Stahler, Mennis, Cotlar, and Baron 2009) suggest that one of the main factors it distance to service.

Therefore Task B focused on the simulation of the impact of infrastructural improvement. Hereby the residence of an individual and the corresponding driving distance to the next hospital was taken into account on NUTS3 level. One sub-scenario considered immediate implementation of the improved service structure and a second one focused on the more realistic concept where it takes three to five years till the necessary changes take full effect.

2.4 Task C: "Diabetes Prevalence and Impact"

Another variable which we studied and has an significant impact on the development of psychiatric diseases and hospitalization in all six countries is diabetes as co-morbidity. Diabetes prevalence is interconnected with psychiatric disorders (Šprah, Dernovšek, Wahlbeck, and Haaramo 2017) and is expected to rise over the next years.

Therefore target of the last scenario was the investigation of the positive correlation between diabetes mellitus and re-hospitalization rate. In order to prove or falsify different expert opinions, stating a 1.5 to 2.5 times higher risk of being re-hospitalized when suffering under diabetes, corresponding sub-scenarios were defined.

Clearly the three tasks are quite constructive and could be fulfilled iteratively extending one simulation model.

3 MODEL DEFINITION

The developed agent-based model can be divided into two parts of which the first one, an agent-based population model, can be seen as the underlying model-framework. It has been developed in the course of a previous project and has already been used for several other applications. It is briefly explained in 3.1. The second part, the disease and hospitalization model, is explained in 3.2 and can be seen as a

superimposed module on top of the population model. The latter was developed specifically for described modelling tasks in the Cephos-Link project.

3.1 GEPOC - An Agent-Based Model of Austria's Population

During the last decades a great number of such so-called population models have been developed all over the world, which, in combination with economic and/or health models, can be used for supporting internal political decisions (e.g. (Andreassen 1993, Kelly 2003, Spielauer 2013)). Most of them are microscopic simulation models wherein the population is no longer simulated as a whole (compare (Malthus 1798)), but as a sum of individuals.

This type of population modeling is usually advantageous due to its flexibility with respect to model-extensions and hence plays an important role in Austria's Comet founded health-care project DEXHELPP (**DEC**ision **S**upport for **HE**a**L**th **P**olicy and **P**lanning). In the course of this project an agent based population model – GEPOC (**GE**neric **PO**pulation **C**oncept) – for Austria was developed, implemented, briefly validated (Bicher, Glock, Miksch, Schneckenreither, and Popper 2015, Miksch, Jahn, Siebert, Glock, Bicher, Schneckenreither, Urach, and Popper 2016) and analyzed (Bicher and Popper 2016) in order to create a solid foundation for generating decision-support models for Austrias health care system.

The model's main features can be summarized as follows:

- The model and is able to simulate Austria's population from 1999 until 2050. The model has been validated by comparison with data and prognosis gained from the Austrian Bureau of Statistics.
- The model is agent-based with two types of agents. First, the model consists of agents that represent real persons and each "person-agent" has a certain age, sex and residence (the latter is a comparably new add-on). Second, the model has one "government-agent" which takes care of actions beyond the behavior of persons – e.g. immigration.
- In principle the model is developed to use that many "person-agents" as real persons, but it may be simulated with a lower resolution as well (1:10,1:100,...).
- The model may validly be called with time-steps of arbitrary length between one hour and one year. A specific routine recalculates transition-probabilities whenever the model's time-step length has been changed. This probably most interesting feature of the model is necessary for its role as a generic framework for health-care models. Note that applications for health-care purposes might use completely different time-scales: Epidemiological applications usually require daily model-updates while long-term intervention strategy analysis is more conveniently modeled with yearly updates due to data availability.
- Each time-step each "person-agent" has a chance to emigrate, die or recreate according to specific age and sex dependent rules. Moreover the "government-agent" creates a number of immigrants each time-step and distributes them according to specific distributions.
- Irregardless of the time-step length, the model takes precise care about dates and durations. That means all actions of an agent that take place in between two time-steps are treated in discrete-event style. This is not standard for ABMs and is very important for agent-agent interactions.
- The model is implemented in Python3. The implementation is parallelized and uses Monte Carlo sampling to randomly generate individuals.

Since the final validation stages of GEPOC in mid 2016 the population model was used in several applications of which most of them are still unpublished. Probably the most successfull one was finished in late 2016 and dealt with the simulation of MMR (Measles, Mumps, Rubella) vaccination-rates in Austria. Results were published by Austrian Ministry of Health and Women's Affairs in their official measles report 2016 (Austrian Ministry of Health and Women's Affairs 2016). Success of this and other projects recommended GEPOC as a generic base-model for the modeling tasks in Cephos-Link.

3.2 Cephos-Link Simulation Model

As interregional comparisons are one main goal of the study the first step of the modeling process considered the extension of the parametrization of GEPOC by Slovenia (about 2 Mio inhabitants) and the region of Veneto in Italy (about 5 Mio inhabitants). The extended model parametrization was validated using the European social statistics 2013 study (short EUROPOP2013) from Eurostat (Kotzeva and Statistical Office of the European Communities 2013).

The modeling process of the hospitalization module itself was done in three phases according to the three defined scenarios. Hereby pooled data for year 2006 in Austria and year 2013 in Slovenia and Veneto, as well as structural knowledge from domain experts was used. We state the main features of the resulting model for each of the modeling tasks.

- The first step of task A considered the introduction of hospitalizations. Each time-step each agent has a certain age and sex dependent contact-probability with a hospital. In case the agent has already had a hospital-stay in a time-period of less than 365 days before, the hospital contact is counted as re-hospitalization otherwise as index-stay or index-hospitalization.
- After being admitted to a hospital, the agent spends some time there. The length of stay is determined by specific distributions dependent on age, sex and whether this is the agent's index stay or not.
- In addition to age, sex each agent has moreover a certain diagnosed diseases state which is either psychotic, non-psychotic or healthy. In case an agent has an index-stay in a hospital, either the first or the second state is assigned. Undiagnosed agents, i.e. agents without contact to a hospital, are always assumed to be healthy. Modeling undiagnosed diseases is, first, not part of the research questions and, second, impossible to parametrize due to unavailable data.
- In case an agent is readmitted to a hospital, time between index-stay and re-hospitalization is also distributed according to age, sex and disease-state dependent distributions.
- In order to model task B each agent is additionally assigned a certain mean driving duration to the closest hospital according to the agent's residence (NUTS3 region). This duration has a more or less inverse proportional influence on the re-hospitalization rate.
- In order to model diabetes mellitus (DM) according to task C, age- and sex- resolved prevalence numbers for 2010 and prognosis for 2030 were collected (open-access resources for the international diabetes atlas (International Diabetes Federation 2015)), preprocessed and used for parametrization of a diabetes module on top of GEPOC. After validation the module was added to the final model from task B. Hereby each agent was added a chance to fall sick with DM dependent on its age and sex. As DM is a progressive disease agents cannot recover.
- Furthermore a diabetes-state dependent factor positively influencing the re-hospitalization rate was introduced.

Accoring to the modelling task we henceforth denote the developed simulation model for task A as model A (model B and C analogously).

3.3 Calibration Problems

While parameter values for model A could be established by preprocessing pooled data, parameters for model B and C are more abstract. The quantitative influence factors of the two agent specific parameters "mean driving distance to the next hospital" and "diabetes-state" on the re-hospitalization probability could not be pooled from data, but was given by domain experts. This lead to the following problem:

In case one simply uses the multiplication of the given re-hospitalization probabilities from model A by the given influence factors as new re-hospitalization parameter in model B or C not only the distribution of re-hospitalizations among the population was changed, which was intended, but also the total value or readmissions, which was highly unwanted. Obviously some **normalization constant** is missing that

norms the influence factors. As these normalization factors cannot be determined from data they have to be calibrated.

The resulting calibration problems can be formalized as follows: Let $\vec{X}$ denote the state vector of an agent $\vec{X} := (\text{age,sex,diagnosis})^T$ and let $p_A(\vec{X})$ stand for the probability for the re-hospitalization of an agent with state-vector $\vec{X}$ in model A. Let moreover $R_A(p_A)$ denote the total number of re-hospitalizations model A results for the index-year of the simulation using parameter function p_A.

Definition 1 (Calibrationproblem A→B) Define $R_B(p_B)$ analogously to $R_A(p_A)$ for model B then we aim to find a normalization constant C_B via

$$C_B := \underset{C\in\mathbb{R}}{\operatorname{argmin}} \left(|\mathbb{E}(R_A(p_A)) - \mathbb{E}(R_B(p_A \cdot f \cdot C))|\right).$$

Hereby f denotes the influence factor of the agent's NUTS3 region.

Definition 2 (Calibrationproblem B→C) Define $R_C(p_C)$ analogously to $R_A(p_A)$ for model C then we aim to find a normalizing constant C_C via

$$C_C := \underset{C\in\mathbb{R}}{\operatorname{argmin}}(|\mathbb{E}(R_A(p_A)) - \mathbb{E}(R_C(p_A \cdot f \cdot C_B \cdot g \cdot C))|).$$

Hereby the notation of Definition 1 was used and g denotes the influence factor of the agent's diabetes state.

Both calibration tasks had to be solved for all three countries and several sub-scenarios with different influence factors each. Summarizing, 15 normalization constants had to be calibrated based on stochastic simulation results. For all of them we faced a scalar stochastic optimization problem. The following chapter gives a rough overview about possible calibration strategies for these applications.

4 STOCHASTIC OPTIMIZATION PROBLEMS

The usual approach to a stochastic optimization problem consists of using a metaheuristic. Metaheuristics make very few assumptions about the model which needs to be calibrated and are therefore applicable to a variety of problems. There are many different metaheuristics which can be classified by various properties. An overview of the most common metaheuristics can be found in (Blum and Roli 2003). A general distinction can be made between single-point and population based search. The most widely used example of a single-point algorithm is simulated annealing (Kirkpatrick, Gelatt, Vecchi, et al. 1983, Černỳ 1985). Genetic algorithms are a commonly used metaheuristic implementing a population based search (Holland 1975). Another important classification for metaheuristics concerns the usage of memory. Memoryless algorithms use only the current state to determine the next step in the search, whereas other algorithms keep track of the entire (or at least the recent) search history. A prominent example of a metaheuristic using memory is the tabu-search (Glover 1989).

Although metaheuristics can be applied to a variety of optimization problems, there is no algorithm whose performance is generally better than the others. A metaheuristic performing above average on one type of problem provides generally worse results on other optimization problems. In fact, the averaged cost for finding a solution for all the problems in a problem class is the same for all solution methods (Wolpert and Macready 1997). Therefore it is crucial to analyze which optimization algorithm is best suited for the problem at hand. Furthermore, there is no guarantee that the algorithm finds a global optimum. For some algorithms, there exist statements of convergence for only a small set of problems (e.g. (Gutjahr 2003) for the convergence of the ant colony optimization). For other algorithms convergence can be proved under specific assumptions which are not met in most practical appliances (e.g. (Hajek 1988) proved the convergence of the simulated annealing under the assumption of a specific cooling schedule and an infinite number of iterations).

5 MODEL ANALYSIS AND NEW CALIBRATION STRATEGY

Tests using a simulated annealing approach quickly turned out to be far too inefficient for the given problem. As most population-based algorithms are optimized for multidimensional calibration problems and require even more simulation executions, we did not even attempt to try any of those. Instead we decided to investigate the stochastic elements of the model in order to see if there was a more straight forward way to determine the desired parameters' optimal values.

For the remainder of this chapter we describe our solution of Calibration Problem A→B (Definition 1) for Austria and index-year 2006 (2006.01.01-2007.01.01). All other calibration problems were treated analogously.

5.1 Stochastic Analysis

Define a discrete random variable $R_i \in \{0,1\}$ indicating whether (1) or not (0) agent i is re-hospitalized during the observed index-year. The probability for readmission strongly depends on the property whether the agent has had an index-stay before. Let random variable I_i be either 1, in case agent i has had an index-stay before, or 0 otherwise, then

$$P(R_i = 1) = P(R_i = 1|I_i = 1)P(I_i = 1) + \underbrace{P(R_i = 1|I_i = 0)}_{=0} P(I_i = 0).$$

The correspondent probabilities $P(R_i = 1|I_i = 1)$ and $P(I_i = 1)$ do not depend on the agent itself, but on a vector of properties $\vec{X}$ such as age, sex and diagnosis. Therefore there exist agent-independent random variables R and I so that

$$P(R_i = 1|I_i = 1)P(I_i = 1) = P(R = 1|I = 1, \vec{X} = \vec{X}_i)P(I = 1|\vec{X} = \vec{X}_i).$$

As a part of task A the numbers

$$p_A(\vec{X}_i) := P(R = 1|I = 1, \vec{X} = \vec{X}_i)$$

were determined for all possible property-combinations of $\vec{X}$.

Defining $R_A(p_A)$ as the total number of re-hospitalizations after the observed time-period, we get

$$R_A(p_A) = \sum_{i=1}^{N} \mathbb{1}_1(R_i) = \sum_{i=1}^{N} R_i, \tag{1}$$

wherein $\mathbb{1}_1(\cdot)$ describes the indicator-function for 1, and

$$\mathbb{E}(R_A(p_A)) = \mathbb{E}\left(\sum_{i=1}^{N} R_i\right) = \sum_{i=1}^{N} \mathbb{E}(R_i) = \sum_{i=1}^{N} P(R_i = 1) = \sum_{i=1}^{N} p_A(\vec{X}_i)P(I = 1|\vec{X} = \vec{X}_i). \tag{2}$$

5.2 Calibration Strategy

In order to fulfill the first calibration task (Definition 1) we are required to match above number with the expected results from model B. Herein

$$P(R_i = 1) = P(R = 1|I = 1, F = F_i, \vec{X} = \vec{X}_i)P(I = 1|\vec{X} = \vec{X}_i)$$

can be observed with F_i being the agent's NUTS3 region. Note that F_i only influences the re-hospitalization probability, but not the probability for its index-hospitalization. By definition of model B

$$P(R = 1|I = 1, F = F_i, \vec{X} = \vec{X}_i) = p_B(\vec{X}_i, F_i) = p_A(\vec{X}_i)f(F_i)C_B,$$

wherein $f(F_i)$ denotes the influence of the NUTS3 region F_i on the probability. Hence

$$\mathbb{E}(R_B(p_A \cdot f \cdot C_B)) = \sum_{i=1}^{N} p_A(\vec{X}_i) f(F_i) C_B P(I = 1 | \vec{X} = \vec{X}_i) = C_B \sum_{i=1}^{N} p_A(\vec{X}_i) f(F_i) P(I = 1 | \vec{X} = \vec{X}_i), \quad (3)$$

which poses the foundation of a very simple new calibration strategy. Using the notation

$$R_A := R_A(p_A), \quad R_B(C_B) := R_B(p_A \cdot f \cdot C_B)$$

we get

$$\mathbb{E}(R_B(C_B)) = C_B \cdot \mathbb{E}(R_B(1)). \quad (4)$$

Thus the normalization constant has a linear influence on the expected value in task B and can precisely be determined comparing the expected value of model A with the expected value of model B using $C_B = 1$. In that case we obtain

$$C_B = \underset{C \in \mathbb{R}}{\operatorname{argmin}}(|\mathbb{E}(R_A) - C \cdot \mathbb{E}(R_B(1))|) \Rightarrow C_B = \frac{\mathbb{E}(R_A)}{\mathbb{E}(R_B(1))}. \quad (5)$$

In case we are able to determine the two expected values $\mathbb{E}(R_A)$ and $\mathbb{E}(R_B(1))$ we can calculate the normalization constant C_B by simple division. The first one is usually impossible but there are very robust ways to approximate them.

5.3 Law of the Iterated Logarithm

Of course the most straight forward approach to estimate the expected value is the arithmetic mean. The law of large numbers guarantees

$$\mathbb{E}(X) = \lim_{M \to \infty} \frac{1}{M} \sum_{j=1}^{M} X_j$$

wherein $X_j, j \in \{1, \ldots, M\}$ describe M independent evaluations of a random variable X with finite variance σ^2. In our case this refers to independent simulations of the model.

As the convergence of stochastic sums is one of the most rigorously researched area of probability theory we are able to give an estimate for the error done, when estimating

$$\mathbb{E}(X) \approx \frac{1}{M} \sum_{j=1}^{M} X_j =: \overline{X} \quad (6)$$

for reasonable large M. Hereby we refer to the Law of the Iterated Logarithm (Hartman and Wintner 1941) which states that the arithmetic mean $\overline{X}$ of M (M reasonably large) independent, equivalently distributed random numbers with finite variance σ^2 almost surely spreads on an interval

$$\left[-\sigma \sqrt{\frac{2\ln(\ln(M))}{M}}, \sigma \sqrt{\frac{2\ln(\ln(M))}{M}} \right] \quad (7)$$

around the actual mean $\mathbb{E}(X)$. The standard deviation in this formula is basically unknown but could be approximated using the sample variance

$$\overline{\sigma} \approx \sqrt{\frac{1}{M-1} \sum_{j=1}^{M} (X_j - \overline{X})^2}. \quad (8)$$

Combining (6), (7) and (8) we not only get a reasonable estimation for $\mathbb{E}(X)$ but also get a robust interval for the error $|\overline{X} - \mathbb{E}(X)|$. In our calibration task the first information is required, but the second one can be very valuable. It can be used to fit the required normalizing constant up to predefined error value. Define

$$\widehat{X} := \overline{\sigma}\sqrt{\frac{2\ln\ln(M)}{M}}$$

for a random variable X and reasonable large M we get

$$\mathbb{E}(R_A) \in \left[\overline{R_A} - \widehat{R_A}, \overline{R_A} + \widehat{R_A}\right], \mathbb{E}(R_B(1)) \in \left[\overline{R_B(1)} - \widehat{R_B(1)}, \overline{R_B(1)} + \widehat{R_B(1)}\right] \tag{9}$$

and therefore

$$C_B \in \left[\frac{\overline{R_A} - \widehat{R_A}}{\overline{R_B(1)} + \widehat{R_B(1)}}, \frac{\overline{R_A} + \widehat{R_A}}{\overline{R_B(1)} - \widehat{R_B(1)}}\right], \tag{10}$$

as all numbers are positive. Using (10) it is possible to approximate the desired parameter value as precisely as necessary applying the following strategy in a loop:

1. Simulate model A.
2. Simulate model B with $C_B = 1$.
3. Update the sample mean $\overline{R_A}$ and the sample standard deviation.
4. Update the sample mean $\overline{R_B(1)}$ and the sample standard deviation.
5. Calculate the bandwidths $\widehat{R_A}$ and $\widehat{R_B(1)}$.
6. If the interval length in (10) is small enough, break the loop and find $C_B := \frac{\overline{R_A}}{\overline{R_B(1)}}$.

This strategy can be interpreted as an error controlled version of the original method. We did not apply this idea for our application yet.

5.4 Calibration Results

Based on $M = 100$ simulation runs of model A we determined

$$E(R_A) \approx \overline{R_A} = 12708.42 \pm 17.71$$

re-hospitalizations in Austria for the year 2006 as a reference value. The uncertainty of 17.71 was the result of the sample standard deviation of about 101.35 and $\sqrt{2\ln(\ln(100))/100} \approx 0.174$. Another $M = 100$ simulation runs of the model B using $C_B = 1$ resulted in

$$E(R_B(1)) \approx \overline{R_B(1)} = 12773.57 \pm 16.50.$$

Altogether

$$C_B = \frac{E(R_A)}{E(R_B(1))} \approx 0.994899, \quad C_B \in [0.9922, 0.9975]$$

could be determined.

It is quite obvious that the obtained confidence interval might still be a little bit large for being convinced about the validity of the parameter's value. Note that for probabilities that close to one even the third decimal might have a major impact on the simulation outcome. Instead of increasing the sample size we chose to look at a different observable: Let I_A and I_B define the total number of index-hospitalizations that result from model A and model B, respectively, then the quotient $Q_B(C) := R_B(C)/I_B$ still satisfies

$$\mathbb{E}(Q_B(C)) = \mathbb{E}\left(\frac{R_B(C)}{I_B}\right) = C \cdot \mathbb{E}\left(\frac{R_B(1)}{I_B}\right) = C \cdot \mathbb{E}(Q_B(1)).$$

As I_A and R_A (such as I_B and $R_B(C)$) are positively correlated (more index-hospitalizations lead to more re-hospitalizations) it is fair to assume that the quality of the confidence interval is improved determining the parameter value with this observable instead. Indeed we obtained

$$C_B = \frac{\mathbb{E}(Q_A)}{\mathbb{E}(Q_B(1))} \approx \frac{\overline{Q_A}}{\overline{Q_B(1)}} = \frac{0.58333}{0.58634} = 0.994867, \quad C_B \in [0.9933, 0.9958],$$

using the same $M = 100$ simulation runs each, which posed a small improvement without increasing the sample size.

As mentioned the calibration problems for the other two countries, sub-scenarios and Calibration Task $B \to C$ was solved analogously. The solutions benefit from the high precision of the determined calibrated value as the resulting compensation parameters lie between 0.891 and 0.999.

6 CONCLUSION

The presented strategy for calibration of a stochastic model is based on the idea of finding a invertible function h and a known p_0 so that

$$E(X(p)) = h(p) \cdot \mathbb{E}(X(p_0)). \tag{11}$$

If such a function is found, only two expected values have to be approximated in order to calibrate the parameter. As additional feature the law of the iterated logarithm gives a measure for the quality of the fit.

In our application it is not a large surprise that the influence of the normalization constant on the mean value is a linear one, but one has to make use of this observation. Only 200 simulation runs were necessary to calibrate a parameter with less than 0.2% uncertainty while a standard stochastic calibration algorithm would have required thousands of simulation executions to achieve the same performance. We gave an example that efforts for formalizing and analyzing (parts of) huge agent-based models with probability theoretic ideas if often time well spent.

ACKNOWLEDGMENTS

- K-Project DEXHELPP is supported by BMVIT, BMWFW and the state of Vienna via COMET - Competence Centers for Excellent Technologies. Programme COMET is processed by FFG. Project number 843550.
- This study was funded as part of the CEPHOS-LINK (Comparative Effectiveness Research on Psychiatric Hospitalisation by Record Linkage of Large Administrative Data Sets) project by the EU Seventh Framework Program 7, grant agreement number 603264.

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